Introduction to Model Validation

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Outline

- Motivation
- Introduction
- Model Validation Steps
- Model Validation Applied to Example Structure
- Conclusions

Motivation

Reasons for validations of mathematical models:

- **Need to replace experiments with model predictions – requires accurate model predictions**
- **Necessity to prove reliability of structure under broad spectrum of environments**
- **System may be undergoing changes in design**

Useful to confirm modelers' abilities to produce accurate models

Introduction

Model validation is the "process of determining the degree to which a computer model is an accurate representation of the real world from the perspective of the intended model applications."

Meanings:

- **Validation - comparisons of model predictions to experimental outcomes**
- **Response measures used in comparisons must reflect what is important in model use**
- **Required accuracy must reflect model use**

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Introduction

Model validation - usually discussed in connection with *verification* and *uncertainty quantification (UQ)*. UQ will be considered, later, but not verification

Verification is the process of determining that a computational model accurately represents the underlying mathematical model and its solution. Two types:

- **Code verification**
- **Solution verification**

Introduction

Groups involved in performance of validation:

- ***System Analysts/Modelers*: Creators of computational models**
- ***Experimentalists*: Planners and performers of calibration and validation experiments**
- ***Validation Analysts*: Performers of validation comparisons**
- ***Customers*: Authorizers of validation analysis**
- ***Stakeholders*: Those with interest in outcome of validation comparison**

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Introduction

This is the first in a sequence of tutorials involving validation of structural dynamic models. The others:

- ***Selection of response features and adequacy criteria***
- ***Uncertainty quantification - probabilistic***
- ***Uncertainty quantification - epistemic***
- ***Model correlation and calibration***
- ***Example of model validation***

Validation - Outline

Validation Planning

- Specify the model use and purpose (What decision is to be made?)
- Specify validation experiments
- Specify the conceptual model
- Specify the mathematical model
- Specify the computational model
- Specify the physical system response measures of interest
- Specify the validation metrics
- Specify the domain of comparison
- Specify the calibration experiments
- Specify the adequacy criteria (validation requirements)

Validation - Outline

Validation Experiments, Predictions, Comparisons

- Perform calibration experiments and calibrate model
- Perform validation experiments and transmit required information to modelers
- Create model and generate model-based predictions
- Perform validation comparisons and judge validity of model
- Take actions regarding use of model

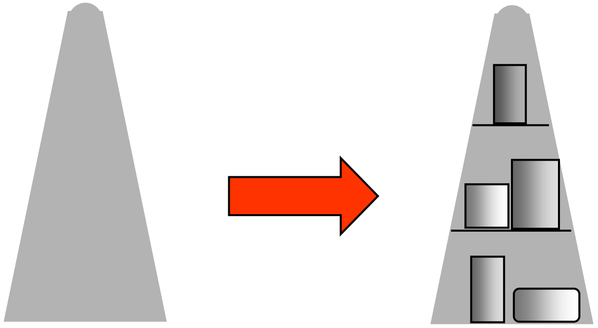
Validation – Specify Model Use and Purpose

Use and purpose:

- **Applications for which model is developed**
- **Current validation may be one in sequence**
- **Compare current system to ultimate system of interest – sufficient accuracy required in current validation to assure desired accuracy in ultimate validation**

Example

- Ultimate system – Shell-payload structure
- External, layered shell & internal comps
- Extern shell – composite bonded to alum
- Current validation – computational model of shell structure



$5.25 \times 10^{-2}\ g^2 / Hz$

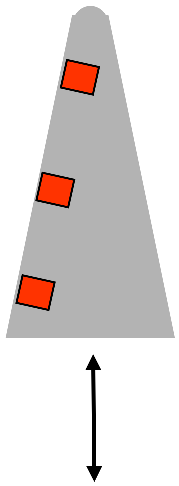
Validation – Specify Validation Experiments

Validation experiment:

- **Experiment designed and performed to generate data for model validation (Different from calibration experiments)**

Example

- Eight nominally identical shell structures subjected to base-excited, stationary random vibration
- Excitation is band-limited white noise, [10,2000] Hz, 5.25x10^{-2} g^2/Hz
- Electrodynamic shaker, three minute duration
- Uniaxial acceleration responses measured in axial direction, at three locations



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Validation – Specify Conceptual Model

Conceptual model of physical system:

- **Assumptions and description of physical processes from which mathematical model and validation experiments can be constructed**

Example

- Shell structures nonlinear – nonlinearity assumed minor
- Ensemble of structures random
  - Materials, geometries, bond thickness, bond adhesion, etc.
  - Only material properties of bonding material assumed random
  - Modulus of elasticity, E, and shear modulus, G, modeled as random
  - All other properties modeled as deterministic

Validation – Specify Mathematical Model

Mathematical model:

- **Mathematical equations, boundary conditions, excitations, initial conditions, and other modeling data**
- **Must include phenomenology anticipated in validation experiments**
- **Must include model for system uncertainty**

Example

- Current structure modeled as linear in specified frequency range and at specified level
- No initial conditions required
- Excitation is stationary random, externally enforced acceleration
- Joint probability model of modulus of elasticity, *E*, and shear modulus, *G*, developed with data from calibration experiments (Both quantities considered spatially constant)

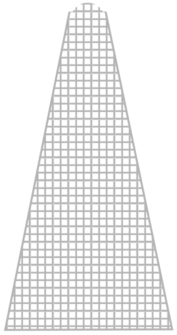
Validation – Specify Computational Model

Computational model:

- **Numerical implementation of mathematical model – numerical discretization, solution algorithm, convergence criterion**
- **Implementation that permits generation of system realizations specified here**

Example

- FEM in Salinas, solid elements
- 1.1 M DOF – “sufficiently” converged
- FEM deterministic – Randomness introduced via Monte Carlo …



$$\boldsymbol{m}\ddot{\boldsymbol{x}} + \boldsymbol{c}\ddot{\boldsymbol{x}} + \boldsymbol{k}\boldsymbol{x} = \boldsymbol{q}$$

$\boldsymbol{m}$	mass matrix
$\boldsymbol{c}$	damping matrix
$\boldsymbol{k}$	stiffness matrix
$\boldsymbol{q}$	force
$\boldsymbol{x}$	response displacement

Dots denote differentiation with respect to time.

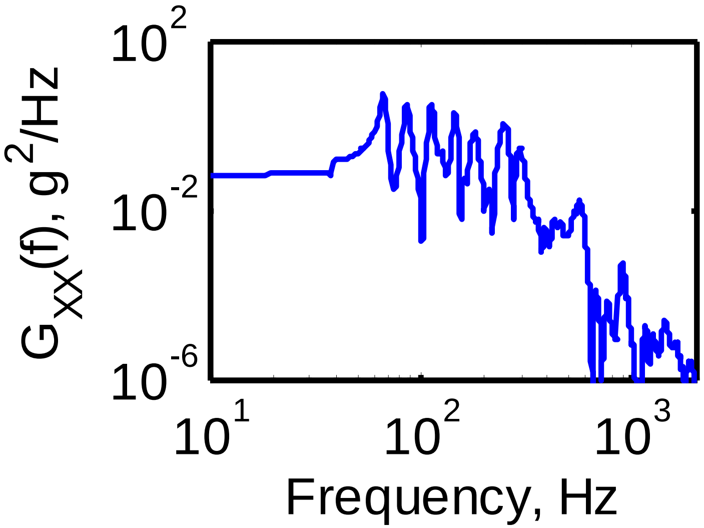
Validation – Specify Response Measures of Interest

Response measure of interest (QOI):

- **Quantities to be used for comparison of model predictions to experimental responses**

Example

Resp accels -> Spectral densities -> Mean Square -> RMS



$$R_1 = \left[\int_0^{300} G_{XX}(f)df \right]^{1/2} = 11.17g$$

$$R_2 = \left[\int_{300}^{2000} G_{XX}(f)df \right]^{1/2} = 2.02g$$

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Validation – Specify Validation Metrics

Validation metric:

- **Precise mathematical means for comparing model-predicted response measures to experimental response measures.**

Example

(To be covered in third and fourth talks.)

Validation – Specify Domain of Comparison

Domain of comparison:

- **Region of environment space and system parameter space where model predictions compared to experiment outcomes**

Example

- Stationary random environments
- Frequencies up to 2000 *Hz*
- RMS excitation amplitudes up to 10.2 *g*
- Exciting structural responses up to approximately 12 *g* RMS

Validation – Specify Calibration Experiments

Calibration experiments:

- **Experiments performed, and used with computational model to infer model parameters (Different from validation experiments)**

Example

- Calibration experiments required to develop joint probability model for modulus of elasticity, E, and shear modulus, G.
- Methods for model development to be covered in third and fourth talks

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Validation – Specify Adequacy Criteria

Adequacy criteria:

- **Values that validation metrics must assume in order for model to be judged valid**

Example

- Accuracy criteria to be covered in third and fourth talks

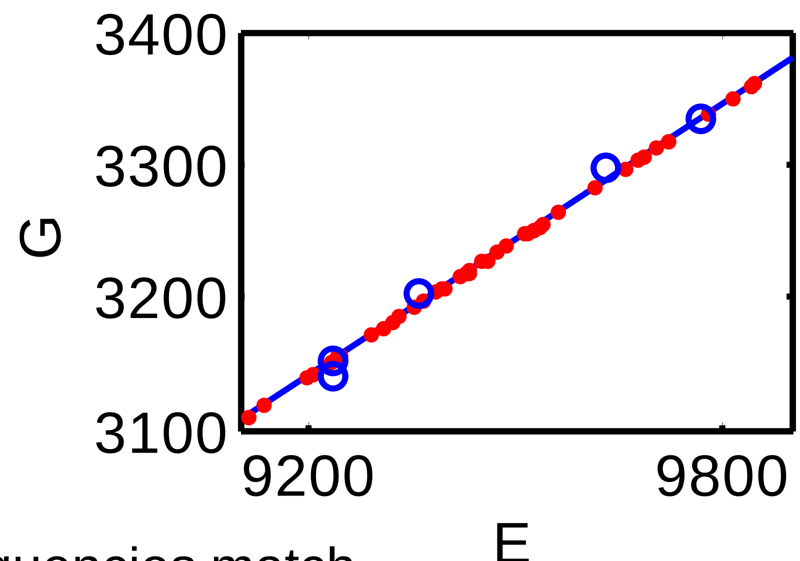
Validation – Perform Calibration Experiments

Calibration experiments:

- **Required to identify material parameters, boundary conditions, laws that govern mechanical joint mechanics, etc.**

Example: *E* and *G*

- Fabricate sandwich structures (5)
- Perform modal tests
- "Measure" modal frequencies



- Develop FEM of sandwich structure
- Compute modal frequencies
- Vary parameters *E* and *G* until model frequencies match experimental frequencies
- Use data to infer probability model (details in third and fourth talks)

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Validation – Perform Validation Experiments

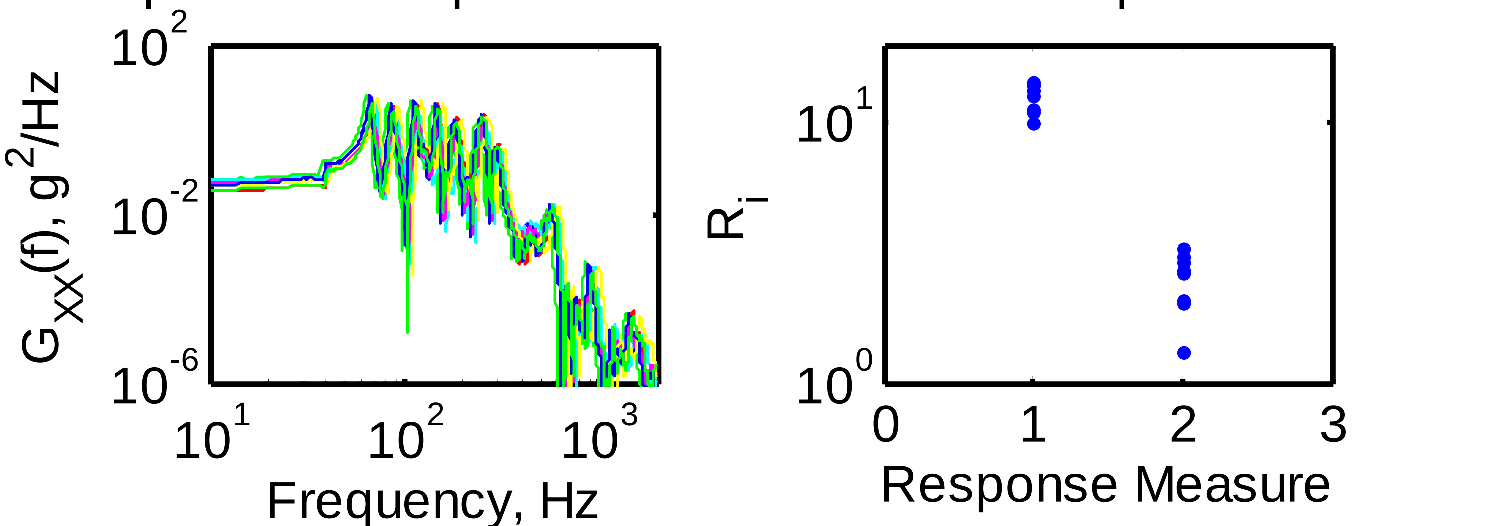
Validation experiments:

- **Perform experiments, measure data, compute response measure to serve as basis for validation of model.**

Example:

Eight structures

Resp accels -> Spectral densities -> Mean Square -> RMS



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Validation – Generate Model Predictions

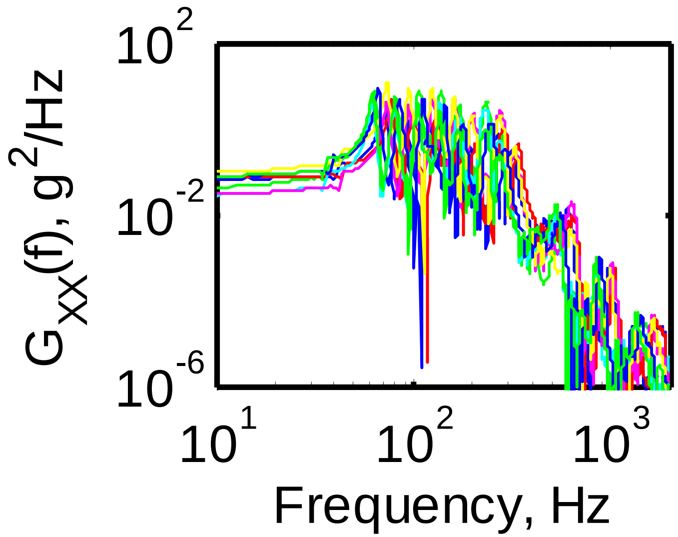
Model predictions:

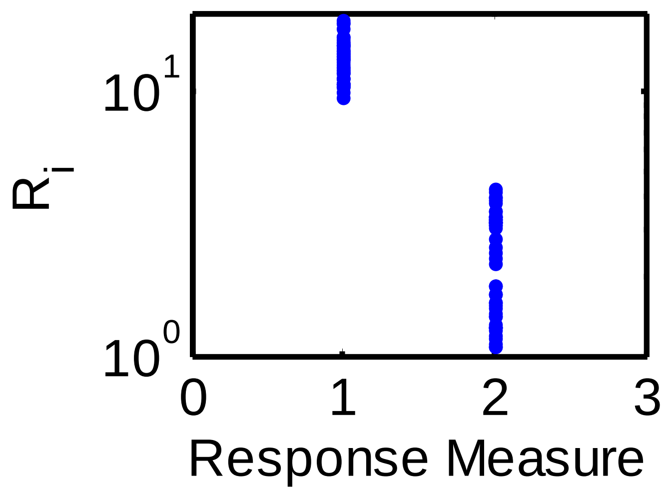
- **Create model, generate predictions of validation results**

Example

Fifty structure models

Resp accels -> Spectral densities -> Mean Square -> RMS





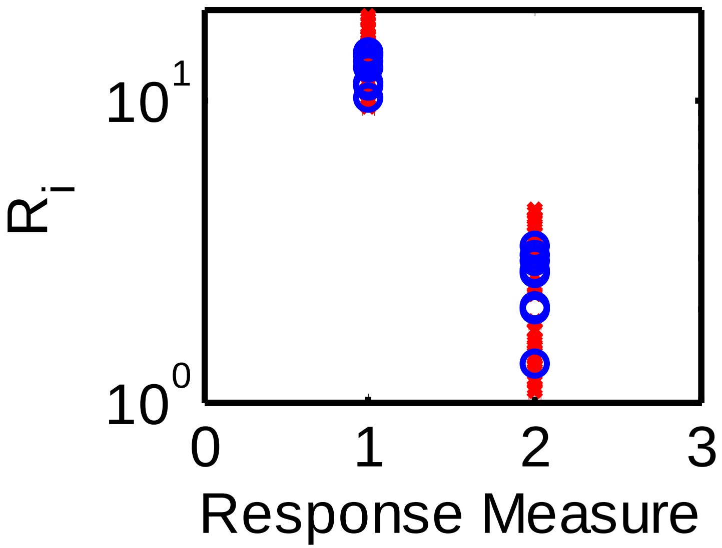
Validation – Perform Validation Comparisons

Validation comparisons and model validity:

- **Compare model-predicted to experimental response measures of interest**
- **Observe to establish whether adequacy criteria satisfied**

Example:

- Use validation metric to compare model-predicted response measures to experimental response measures
- (These response measures match well)



Validation – Take Actions

Actions following validation comparison:

When model not validated:

- Model unsatisfactory
- Unmodeled randomness
- Insufficient physics
- Inadequate data
- Response measure too difficult
- Adequacy criteria too stringent

When model validated:

- Use as basis for next development step
- Make predictions of structural response

Conclusions

- An approach to validation presented
- Includes numerous validation planning steps
- Followed by
  - Testing – Calibration and Validation
  - Model development and prediction
  - Model-to-experiment comparison
  - Actions