Model Correlation and Calibration

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Model Validation History in Structural Dynamics

In the 80's

Validation = Correlation = Updating = Refinement

= "better match" between analysis and test data

In the 90's the definition of validation began to be refined.

One often-referenced definition that has emerged is:

"*Model validation* is the process of determining the degree to which a computer model is an accurate representation of the real world from the perspective of the intended model application." [1]

Here we take the approach that the model will make a blind prediction that will be compared to test results. Pre-determined bounds are set to determine if the model is valid or invalid. The implication is that the validation data are not used for correlation or calibration. But correlation and calibration are important steps toward model validation.

Steps to Model Validation [2]

- Preliminary Steps
  - –Specify model use/purpose (what decision is to be made)
  - –Specify response measures (what the model predicts)
  - –Specify validation features and metrics and comparison domain
  - –Specify calibration experiments
  - –Specify validation experiments
  - –Specify adequacy criteria
- Perform calibration experiments/Calibrate model parameters
- Validation
  - –Perform experiment
  - –Make predictions
  - –Calculate metrics/compare with adequacy criterion
- Subsequent Action
  - –Not valid – Reformulate model/Additional calibration
  - –Valid – Make Predictions

We focus on the step highlighted in blue and expand it to include correlation and calibration.

Model Correlation

- In structural dynamics, originally correlation was the process of finding comparable mode shapes/natural frequencies between eigenvalue analysis of a FE model and modal test data, which will be our context here.
- Correlation is the process where one gains modeling insight by observing differences in comparable quantities between model and test. Some of the best correlation *exercises* are with modal tests of subsystems
- Attempt to correlate the shapes by eye and observe the difference in shapes and associated frequencies. (Mathematical correlations give no insight into what may cause differences). FE analyst and experimentalist should work together to achieve greatest insight.
- Look for evidence of these errors during correlation
  - Faulty element connectivity
  - Faulty boundary conditions
  - Lack of convergence
  - Errors/over-simplification in model form, element choice, defeaturing
  - Typographical errors, for example in modulus
  - Possible parameter errors (leave this last – addressed later)

Model Calibration

- After non-parametric errors have been corrected, calibration of specific uncertain parameters can be valuable.
- It is best to have an experiment and hardware that allows focus on the parameters of interest, if they are known. (Not the final validation experiment)
- Sometimes analysts utilize a "cut and try" approach to calibrate parameters, but a systematic sensitivity approach was developed a long time ago.

$$\Delta\bar{\mathbf{f}} = \mathbf{S}\Delta\bar{\mathbf{p}} \qquad (1)$$

Where

$\Delta\bar{\mathbf{f}}$ = *frequency of experiment – frequency of model*

$\mathbf{S}$ = *matrix of sensitivities of each frequency to each parameter*

$\Delta\bar{\mathbf{p}}$ = *change in parameter to achieve the desired test frequency*

Each sensitivity value is normalized as $(\Delta f \times p)/(f \times \Delta p)$

Sensitivity Equation Setup

Generally equation (1) is overdetermined with more frequencies than parameters and is solved for parameter changes as

$$\Delta \bar{\mathbf{p}} = \mathbf{S}^{+} \Delta \bar{\mathbf{f}} \qquad (2)$$

Bad results may come from this seemingly simple equation if:

- The frequencies are not sensitive to the parameters
- The frequency errors are caused by model form errors
- The frequency errors are caused by parameters not included in (2)
- The model is not already “near” the true solution

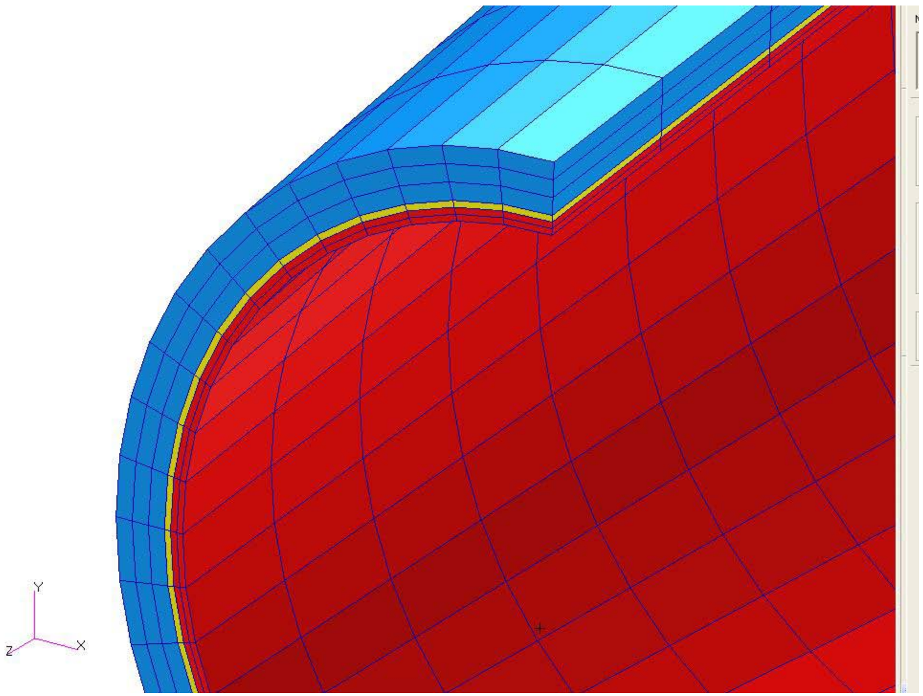
How do we know if we have selected appropriate parameters or not?

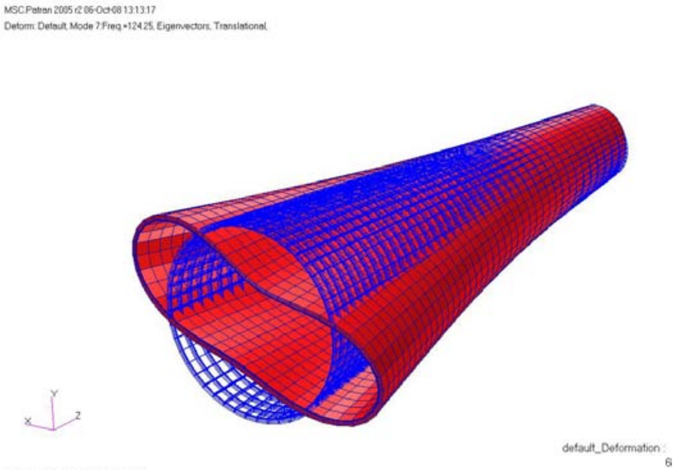
- Tool 1 – Sensitivity matrix rank
- Tool 2 – Parameter correlation
- Tool 3 – Statistically significant parameters

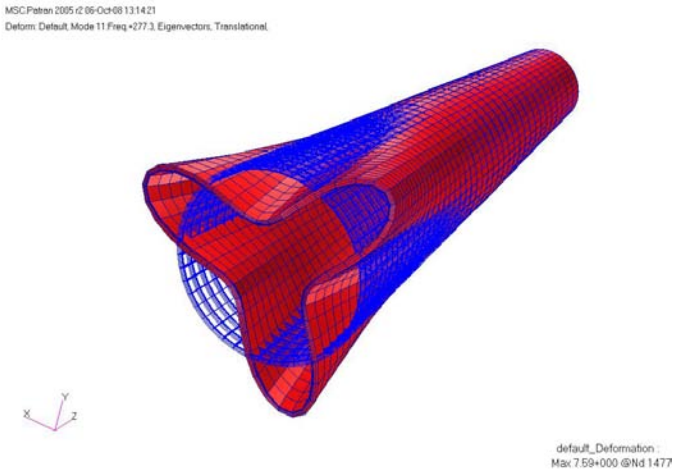
Let’s look at these tools with an example problem

Example problem

A conical shell structure with three material layers is modeled with finite elements. All three layers are modeled with isotropic material properties, parameterized with modulus of elasticity and Poisson's ratio







Example problem

Table 1 shows the initial frequencies from the FE model (based on initial parameter estimates) and modal test with the differences

Table 1 - Initial Model Frequencies Vs Modal Test

FE Frequency	Modal Test Frequency	% Error
124.2	149.3	-16.8%
268.1	318.6	-15.9%
277.3	342.6	-19.1%
442	537.6	-17.8%
461.1	574.8	-19.8%
470.7	559.6	-15.9%
641.3	675.9	-5.1%

Blind Calibration of 6 parameters

Equation (2) was used to solve for all 6 parameters, 3 E and 3 nu and achieved a frequency error of less than one percent for every frequency. Table 2 shows the errors in the parameters before and after the calibration.

Table 2 – Parameter errors before and after blind calibration

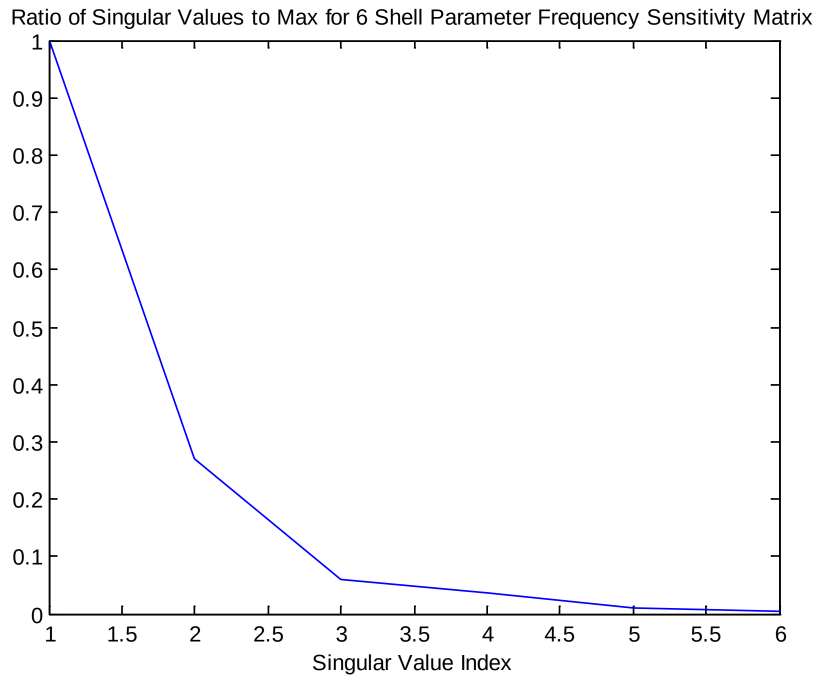
Parameter	Initial % Error	% Error after Calibration
E outer	-20.0000	26.5776
nu outer	0	-35.0651
E mid	-65.0000	-39.7927
nu mid	-33.3333	-34.9270
E inner	2.0408	-33.8752
nu inner	0	-68.4929

The parameter errors are almost all worse after the calibration than before!!! Why?

Tool 1 – Sensitivity Matrix Rank

By looking at the singular value decomposition of the sensitivity matrix, one may see the maximum number of parameters that may be adjusted

Plot of Singular Values from 7x6 Sensitivity Matrix



In the author’s experience, when the singular values drop below about 5% of the maximum, it is of no value to calibrate additional parameters. Here, a maximum of two parameters might be calibrated.

Tool 2 – Parameter Correlation

By looking at the Pearson product-moment correlation coefficient between each pair of sensitivity column vectors, it may be observed that some parameters affect frequencies in about the same way.

$$r_{ij} = \frac{(S_i - \bar{s}_i)^T (S_j - \bar{s}_j)}{\sqrt{(S_i - \bar{s}_i)^T (S_i - \bar{s}_i)(S_j - \bar{s}_j)^T (S_j - \bar{s}_j)}}$$

Table 3 – Correlation of Frequency Sensitivity Among Parameters

	E outer	nu outer	E mid	nu mid	E inner	nu inner
E outer	1	.63	.51	-.55	-.67	-.06
nu outer		1	.60	-.54	-.67	-.38
E mid			1	-.97	-.98	.46
nu mid				1	.96	-.45
E inner					1	-.39
nu inner						1

In the case of highly correlated sensitivity vectors (near 1), the most sensitive parameter may dominate the calibration, even if it is not in error.

Tool 3 – Statistical Significance

A test of significance compares the required parameter change with the estimated noise propagated to the parameter as a standard error of the mean of the parameter.[4]

$$\sigma_f^2 = \Delta\bar{\mathbf{f}}^T \Delta\bar{\mathbf{f}} / (m - n)$$

$$COV\Delta\bar{\mathbf{p}} = \sigma_f^2 [S^T S]^{-1}$$

$$\bar{\sigma}_{mean}(\Delta\mathbf{p}) = sqrt(\mathbf{diag}(COV\Delta\bar{\mathbf{p}}) / m)$$

$$z_i = \Delta p_i / \sigma_{mean}(p_i)$$

If the z score is a big number (approximately >2) then the parameter change required is significant – it is out of the noise.[3]

Tool 3 – SSP continued

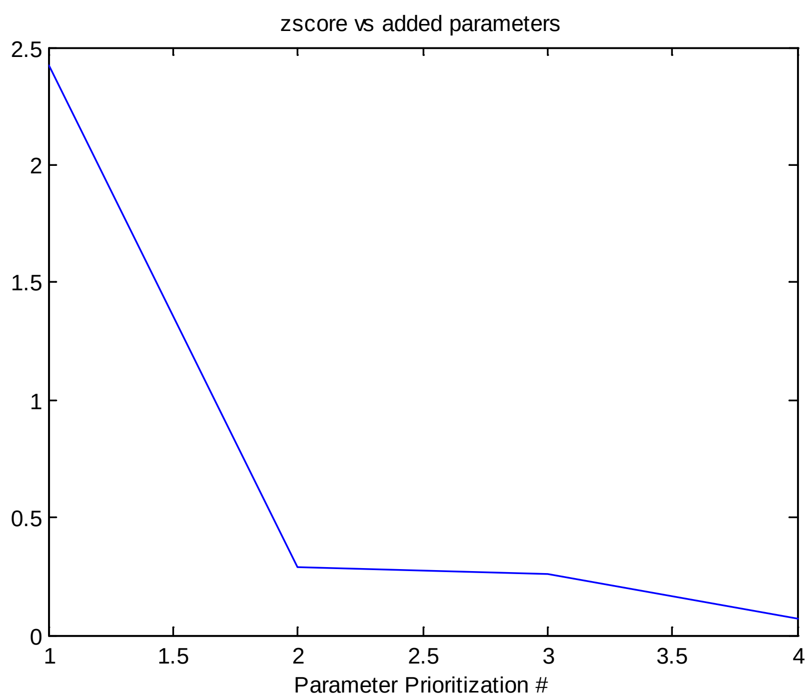
Do not calculate the z scores with all parameters of the sensitivity matrix, because correlation of columns dilutes the z score. Instead

- Calculate the z score for each parameter using only one column of the sensitivity matrix – the highest z score is the most significant parameter
- Now perform the z score calculations using the column for the first parameter and adding one other parameter. Repeat for all other parameters – the highest z score for the remaining parameters is second most important
- Continue adding one column to the sensitivity matrix as above until all parameters have been prioritized.
- Plot the z-score vs prioritized parameters

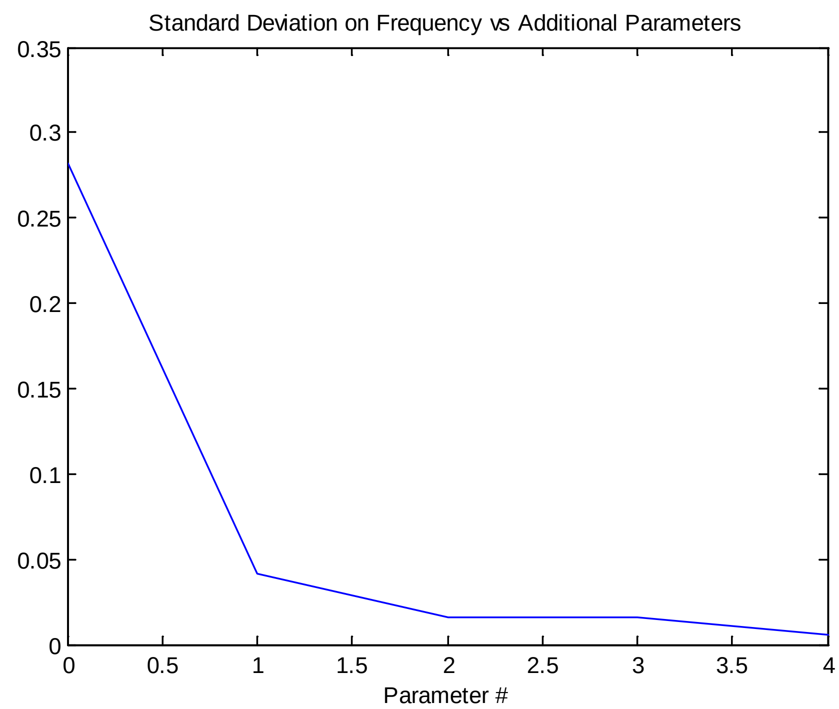
Tool 3 – SSP continued

Z Scores and frequency improvements from the shell example

Z Scores vs # parameters



Std deviation frequency vs # parameters



Tool 3 – SSP continued

Estimated Parameter Change Percentage As A Function of Added Parameters

Parameter	1	2	3	4	True
E outer		25	26	58	25
nu outer					0
E mid	148	101	74	51	186
nu mid			-145	-51	50
E inner				-33	-2
nu inner					0

For this problem, only the first parameter should be estimated (E mid) and then the FE model should be run again and the sensitivity analysis and use of tools repeated. If the sensitivity svd remains similar, a maximum of two parameters could be reasonably calibrated.

Conclusions

- Correlation exercises the model to provide insight to:
  - Debug the model (this should be the focus)
  - Provide evidence for parameters that may need calibration
- Calibration can be performed with data from calibration experiments using sensitivity analysis
- There are limits to accurate calibration of parameters as shown with three tools
  - Sensitivity Matrix Rank (Plotting SVD of sensitivity matrix)
  - Correlation of parameters (Correlation of columns of sensitivity matrix)
  - Statistically Significant Parameters (Z score shows which parameters are out of the noise)

References

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2. Urbina, A., Paez, T.L.,Rutherford, B.,O'Gorman, C., Hinnerichs, T., Hunter, P., "Validation of Mathematical Models: An Overview of the Process", Proceedings of the 2005 SEM Conference and Exposition on Experimental and Applied Mechanics, Paper 210, June 2005

3. Mayes, R.L., *A Tool to Identify Parameter Errors in Finite Element Models,* Proceedings of the 15th International Modal Analysis Conference, Orlando, FL., pp.825-831, February 1997

4. Branham, Richard L., Jr., *Scientific Data Analysis, An Introduction to Overdetermined Systems*, Springer Verlag, New York, pp. 93-95, 1990.