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A Structural Dynamics Model Validation Example with Actual Hardware

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NOMENCLATURE

FE	Finite Element
FRF	Frequency Response Function

ABSTRACT

This paper reports the results of a validation effort of a finite element structural dynamics model of a layered shell structure comprised of a metal layer, a glue layer and a fiber-reinforced composite layer. A validation plan linking requirements to response features through adequacy criteria was established before the structural analysis began. Uncertainty quantification was performed to determine whether dimensional tolerances were important, whether isotropic formulations could be used instead of orthotropic and to compare modal solutions from two different finite element analysis codes. Correlation efforts identified some errors in the model form as well as a calibration error. Significant uncertainty on some material properties was reduced by calibration experiments providing some indication of the spread of those properties. Confidence results from a blind prediction that meets validation adequacy criteria.

1.0 INTRODUCTION

"Model validation is the process of determining the degree to which a computer model is an accurate representation of the real world from the perspective of the intended model application" according to one definition [1]. A case study is provided in this paper for the validation of a finite element (FE) modeling methodology for a three layered conical shell structure. The inner layer is a metal. The middle layer is a filled rubber that acts as a glue to connect the inner layer to the outer layer. The outer layer is a fiber-reinforced composite that is orthotropic. The proposed FE methodology simplified all layers with isotropic material properties. Twenty node quadratic solid elements were utilized. The initial assumption was that there would be three elements through the outer layer thickness, one element through the mid layer thickness and two elements through the inner layer thickness. The engineering question to be answered was whether these assumptions were adequate for modeling this shell structure for certain specific environments. Figure 1 shows a cross section of the layers in a FE model for a structure similar to the one analyzed.

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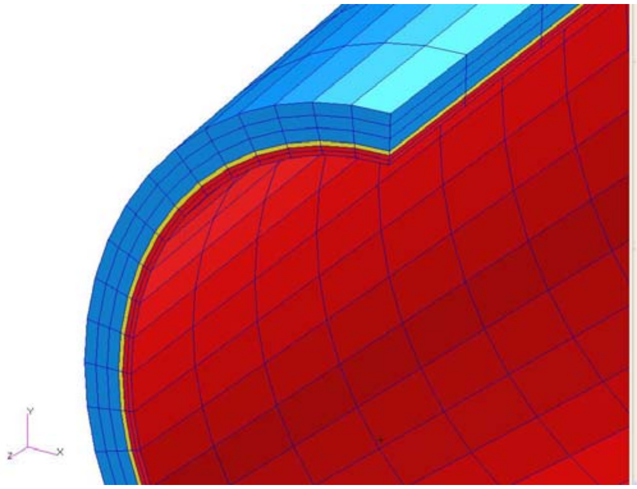

Figure 1 - FE Mesh for Three Layered Shell

A validation plan was developed in advance of a blind prediction by the FE model of response features that could be compared with experimental data from actual hardware. The purpose of the FE model was to be able to couple dynamic loads that would come through the shell to FE models of components to predict their shock or vibration response. Engineers knew from the types of environments considered that certain physics of the shell needed to be exercised appropriately. The requirements were defined to exercise these physics.

Response features that would capture the physics were established based on modal frequencies of a conical shell with free boundary conditions. Each modal frequency was associated with a particular mode shape chosen specifically because it exercised the required physics, i.e. shear, extension, bending, etc. Adequacy criteria were established with consideration for the dynamic effects of other subassemblies that would modify the stiffness and mass of the full system.

A model was compared with a correlation experiment. This was not the same model or hardware that was used for the final validation, but the mesh utilized the same model form. The correlation experiment provided responses that would exercise similar physics and was designed to help uncover unintentional errors before the final blind validation prediction. In the correlation process uncertainty quantification due to several sources was investigated strictly with the model including isotropic uncertainty and code to code variability. Model form and parameter values were to be confirmed in comparisons with the correlation experiment.

A calibration experiment with multiple samples was performed with the purpose of determining the modulus of elasticity and Poisson's ratio for the middle layer. FE models of the calibration experiment utilized sensitivity analysis to infer these parameters and some estimate of the spread in the parameters was estimated.

A validation experiment was developed to provide response features (modal frequencies) for modes that exercised the required physics. A blind prediction was to be made by the FE model after the calibration and correlation exercises. Adequacy criteria that were decided in advance would be utilized to compare the FE model response features with the validation experiment.

The description of the validation process described above is based on following the steps to validation listed below which are given by Urbina, Paez, et al [2].

- Preliminary Steps
    - Specify model use/purpose (what decision is to be made)
    - Specify response measures (what the model predicts)

    - Specify validation features and metrics and comparison domain
    - Specify calibration experiments
    - Specify validation experiments
    - Specify adequacy criteria
- Perform calibration experiments/Calibrate model parameters
- Validation
    - Perform experiment
    - Make predictions
    - Calculate metrics/compare with adequacy criterion
- Subsequent Action
    - Not valid – Reformulate model/Additional calibration
    - Valid – Make predictions

As part of the calibration step above, in addition to calibration, the authors also included another set of exercises which is denoted as "correlation" in this paper. The correlation process included some model based uncertainty quantification as well as predictions for modal testing on non-validation hardware to confirm model form and parameters. The correlation exercise was not used to calibrate any parameters.

In this work, the validation team consisted of several FE analysts, an experimentalist, the customer and an uncertainty quantification subject matter expert. Validation decisions and plans were made in advance of the validation experiment and validation predictions.

2.0 PURPOSE AND REQUIREMENTS

Based on the definition of model validation given in section 1.0, one must know the "intended application" of the model, which might also be called the purpose of the model. From this purpose, the model validation team must establish requirements that can ultimately be related to some response feature through adequacy criteria. The process is discussed elsewhere more fully by Mayes [3].

The model validation team agreed that the purpose of the model was to show that modeling the three layered shell structure with isotropic material property assumptions and six twenty node solid elements through the shell thickness was adequate for certain known environments the structure would experience. From previous experience with these environments on other structures the requirements were set down in terms of motion that would exercise the important physics in the shell. These physics were associated with axial motion, longitudinal bending, shearing in the mid layer in the circumferential direction and uniform stretching in the circumferential direction as would be expected in a breathing mode. These requirements were ultimately related to free modes of vibration of a shell that exercised these motions.

3.0 RESPONSE FEATURES AND ADEQUACY CRITERIA

The response features were chosen to be the frequencies of specific modes of vibration of a free shell structure that exercised the physical motion described in section 2.0. The value of using frequencies instead of shapes is that the frequency is a single quantity that relates directly to stiffness. All the physics of interest affect the stiffness of the chosen modes. The modes whose frequencies exercised the physics are given in Table 1. The convention for the ovaling modes is based on describing motions of a cylinder as the M,N mode where M is the number of sine waves in the radial displacement around the circumference and N is the number of nodes along the longitudinal axis. A breathing mode has uniform radial displacement and is given as M=0.

Table 1 - Modes Associated with Physics

Physics	Mode Description
Axial Motion	First Axial
Longitudinal Bending	First Bending
Circumferential Shear Mid Layer	2,0 Ovaling
Circumferential Extension	0,0 Breathing

The validation experiment hardware that was chosen was basically a cone shaped shell with a few modifications near the closed end. The validation involved a modal test of this structure with enough instrumentation to identify the modes in Table 1. The frequencies associated with these modes were chosen as the response features. Assuming that the mass is easily known, the frequencies of these modes are indicative of the stiffness of the structure for the physics of interest. This rationale establishes the relationship between the response features and the requirements.

The adequacy criteria must establish acceptable model accuracy. Adequacy criteria should be as lenient as possible so that the model does not fail the validation when it is really still useful. Another way to ask the adequacy question is "How poor can the model prediction be and still be acceptable?" The tendency for analysts and experimentalists is sometimes to be too restrictive with the adequacy criterion.

Assuming that the mass distribution is correct, a small frequency error of X percent in the model would indicate that there was a global stiffness error of about 2X percent for a particular mode. The chosen modes were designed to exercise specific important physics. In this case, the validation team agreed that a twenty percent systematic stiffness error would be marginal. Part of the reason that this large an error was acceptable is that the shell structure is connected to other subsystems in the operational environments, which have a large impact on the global stiffness. A ten percent frequency error would correspond to about a twenty percent stiffness difference in a single mode. In addition, significant model uncertainty expressed as a percentage at the 95 percent confidence level, U_m, which was identified, would be added to the ten percent limit. Therefore, for the first axial, 2,0 ovaling, first bending and breathing mode of the free-free conical validation shell, the adequacy criterion was set at $\pm(10^2 + U_m{}^2)^{1/2}$ percent of the modal test frequency. An additional criterion, designed to eliminate the possibility that the main criteria were not satisfied by random chance, was that the lowest four lobed ovaling mode should be predicted within ±15 percent of the modal test frequency.

4.0 CORRELATION EXPERIMENT

A separate correlation experiment was performed on another shell which was a conical frustum with several additional masses and flanges. Although additional uncertainty might be introduced with these additional details, the team decided that this hardware was adequate for the correlation and debugging exercises before blind predictions of the validation conical shell modes were made. Impact testing was performed with the 1/4 pound hammer with the white plastic tip. There were six rings of triax accelerometers with a location at every 90 degrees around the circumference on the outside of the conical frustum. Copper tape was applied before the triax blocks were mounted. Data were collected from eight different impact locations. The SMAC [4] modal extraction package was utilized to extract modal parameters using real mode shapes.
Table **2** lists the natural frequencies and a description of the modes. Where two modes have the same description, one is the orthogonal pair with the other. It is believed by the modal test engineer that all the first 14 elastic modes were extracted. However beyond that frequency, not every mode was extracted, but some additional well excited modes are included with * for mode number.

Table 2- Conical Frustum Experimental Modal Frequencies for Correlation Exercises

Mode #	Frequency (Hz)	Description
1	198.8	2,1 Node Forward
2	208.2	2,1 Node Forward
3	284.7	2,1 Node Aft
4	292.8	2,1 Node Aft
5	510.4	Aft 3 lobe
6	520.8	2,2
7	529.7	2,2
8	561.5	Aft 3 lobe
9	619.4	Mid 3 lobe
10	625.6	Mid 3 lobe
11	799.2	3,2
12	807.8	3,2
13	921.0	Aft 4 lobe
14	943.1	Aft 4 lobe
*	979.7	First bend Y
*	1007.8	First bend Z

5.0 CALIBRATION EXPERIMENT AND RESULTS

The most uncertain properties were the Young's modulus and Poisson's ratio of the middle layer. Various reports listed the Young's modulus at 3.5, 4.0 and 25 thousand psi. Only one value for Poisson's ratio was reported at .456, but there was no documentation as to the origin or reliability of that value. There was speculation that the Poisson's ratio could be as high as .49 for this rubber-like layer. Consequently, it was decided to obtain the components and mix a formulation of the glue to attach steel discs together as shown in Figure 2. Three different nominal thicknesses of the material were utilized, 0.5 inches, 0.060 inches and 0.030 inches. Five sets were made for the largest thickness, and ten sets of each for the two smaller thicknesses. Due to processing difficulties, the two sets of thinner bonds had significant problems with air bubbles in the mix, creating significant spread in the results. Therefore, only the 0.5 inch samples were utilized in the calibration. An accelerometer was placed on one disc in the radial direction and five radial hammer taps on the disc provided frequency response functions (FRFs) from which the shear mode frequency was extracted. Then the accelerometer was moved to the middle of the disc in the axial direction, and five axial hammer taps provided FRFs from which the axial mode frequency was extracted.

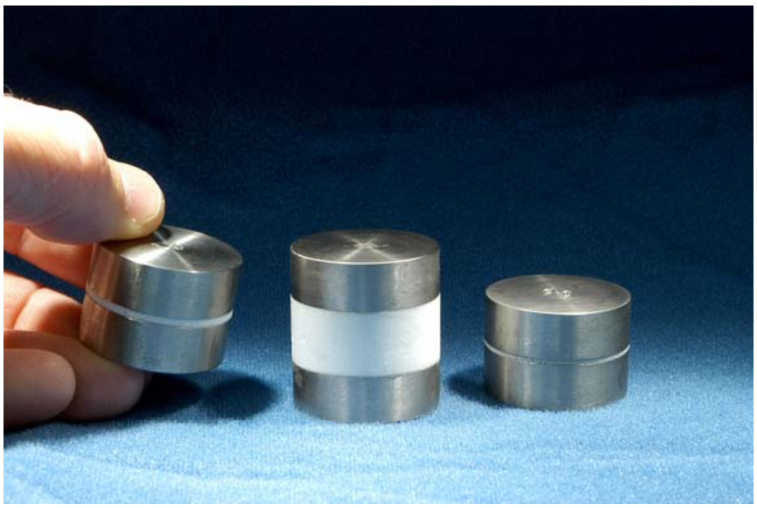

Figure 2 - Calibration Test Hardware for Young's Modulus and Poisson's Ratio

A FE model for the 0.5 inch thick disc samples was generated and an initial estimate of Young's modulus and Poisson's ration was made. Then these two parameters were tuned to obtain frequency matches for the shear and axial mode of the disc system. Figure 3 shows the shear and axial mode shapes from the FE model.

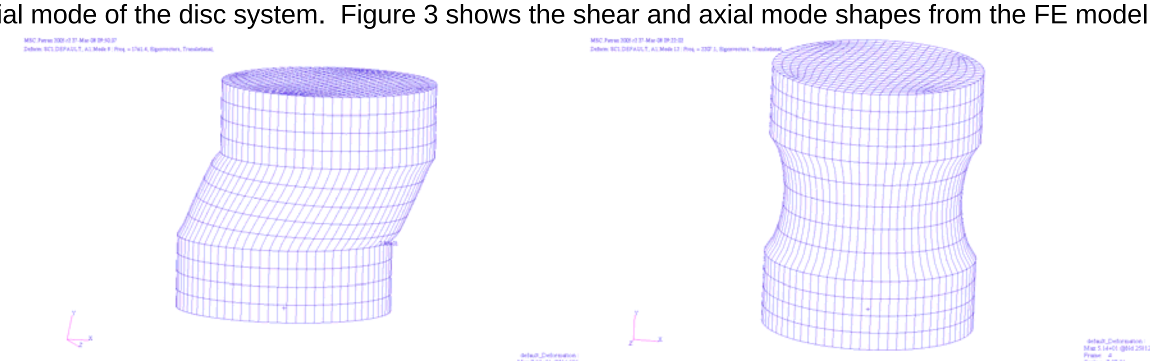

Figure 3 - Shear and Axial Mode Shapes for Calibration Hardware

The final results are given in Table 3 for the Young's modulus and Poisson's ratio after tuning the model parameters to match the test frequencies. The average Young's modulus was 9446 psi with a standard deviation of 242 psi, and the average Poisson's ratio was 0.464 with a standard deviation of 0.004. It is interesting to note that this was the final calibration result, but there was a communication error in the initial calibration. The diameter of the sample was mistakenly used as the radius in the FE model. This provided erroneous values of

13,000 psi and 0.18 for Young's modulus and Poisson's ratio. This was carried into the correlation and will be discussed later.

Table 3 - Calibration of Bond Young's Modulus and Poisson's Ratio

Test Shear Frequency (Hz)	Test Axial Frequency (Hz)	Model Shear Frequency (Hz)	Model Axial Frequency (Hz)	Modulus	Poisson's Ratio
1750	2210	1749	2209	9235	0.465
1800	2275	1799	2272	9770	0.465
1787	2240	1788	2240	9630	0.46
1762	2215	1762	2214	9362	0.462
1750	2230	1750	2227	9235	0.47

6.0 CORRELATION EXERCISES

Several correlation exercises were performed before the final blind validation prediction was made. Some uncertainty quantification studies were performed by just varying certain modeling assumptions. Finally the correlation model was used to predict the results from the correlation hardware modal test.

6.1 Model Based Uncertainty Quantification Studies

Eigenvalue analyses for several variations were performed, and the frequencies of certain key modes were compared with extremes on the variations. The differences in frequency are summarized in the following paragraphs.

Specifications provided tolerances on the shell manufacturing dimensions. The minimum and maximum thickness values were modeled and the frequency differences for the extremes were less than one percent, providing evidence that manufacturing tolerances were not a major effect.

Specifications on allowable voids in the bond were used to model various allowable voids in the model. Frequency differences from the nominal full bond were less than one percent providing evidence that bond voids were not a major effect.

A model with orthotropic material properties for the outer layer was compared with a model utilizing isotropic properties for the outer layer. The frequency differences were less than 0.5 percent for the first ten elastic modes showing that the isotropic simplification was adequate for this work.

A code to code comparison was made for the first ten elastic modes utilizing Sandia National Laboratories' Salinas code and MSC/Nastran. The frequency differences were less than 0.03 percent. This is not true code verification, but helps show that at least the equations are being solved with very similar results from two independent codes operating with the same mesh and material properties and element types.

6.2 Model Correlation Predictions Compared with Modal Test Results for Correlation Hardware

The correlation process (on hardware that was not the final validation hardware) was designed to uncover unintended errors or poor assumptions in the model form that could negatively impact the final validation prediction. The model of the shell frustum was utilized to predict free modes of the hardware. In Figure 4 are shown some typical ovaling mode shapes. In Table 4 are given the initial comparisons between the FE model of the conical frustum and the modal test of the conical frustum. Several frequency differences are greater than ten percent for ovaling modes. In particular, the first ovaling mode is more than twelve percent high in the model. These initial results indicated some unknown errors to the validation team, and additional exercises were performed in the correlation process to attempt to uncover unknown errors. One exercise found that the modulus of the outer layer had to be reduced by more than 50 percent (which was unreasonable based on measured properties) to achieve a match with the first mode, indicating that the problem must be in the other layers.

A decision was made to reinvestigate the calibration of the properties of the bond layer. In that process, the error described at the end of section 5.0 in which the diameter of the FE model was off by a factor of two was uncovered. When this was corrected, it changed the mid layer properties significantly.

Another investigation was performed to examine the defeaturing of the FE model as compared with the actual correlation frustum hardware. It was found that some slots in some internal ribs had not been modeled in the FE model. These slots were added to the model. After this change and the change to the mid layer material properties were made, the frequency errors for ovaling modes in the frustum were reduced. The error in the first ovaling mode was reduced below four percent, and the first bending mode errors were less than three percent which was acceptable to the model validation team.

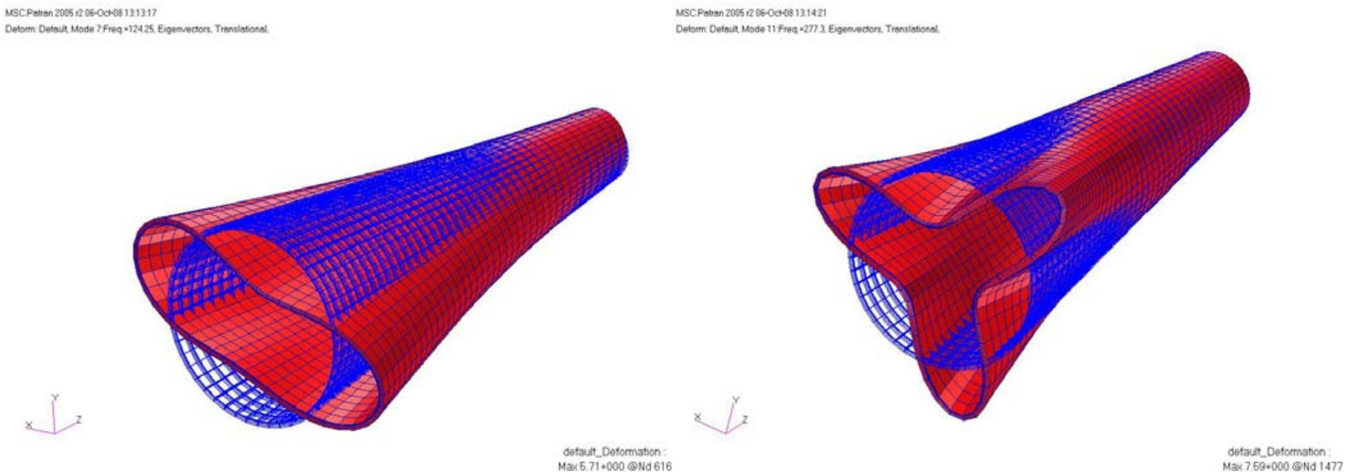

Figure 4 - Ovaling Modes of the Shell Structure

Table 4 - Initial Correlation for Frustum

Mode #	Test Frequency (Hz)	Model Frequency (Hz)	Description
1	198.8	222.4	2,1 Node Forward
2	208.2	233.2	2,1 Node Forward
3	284.7	294.1	2,1 Node Aft
4	292.8	296.2	2,1 Node Aft
5	510.4	592.0	Aft 3 lobe
6	520.8	540.1	2,2
7	529.7	542.5	2,2
8	561.5	639.4	Aft 3 lobe
9	619.4	660.2	Mid 3 lobe
10	625.6	664.1	Mid 3 lobe
11	799.2	802.6	3,2
12	807.8	810.3	3,2
13	921.0	1067.6	Aft 4 lobe
14	943.1	1097.6	Aft 4 lobe
*	979.7	984.4	First bend Y
*	1007.8	970.8	First bend Z

7.0 VALIDATION EXPERIMENT AND BLIND PREDICTION COMPARISON RESULTS

The validation modal test was performed on a conical three layered shell with a closed end instead of a frustum. Impact testing was performed with a 1/4 pound hammer with the white plastic tip. There were three rings of triax accelerometers with a location at every 90 degrees around the circumference on the outside of the aeroshell and one triax at the tip. Data were collected from seven different impact locations. The SMAC modal extraction package was utilized to extract real mode shapes. Simmermacher and Mayes [5] showed for an analytical case that SMAC had no discernable bias errors and the maximum random error in the modal frequency estimate was about three times the specified convergence tolerance with the majority of estimates falling within the convergence tolerance. The default convergence tolerance of 0.05 percent was utilized in this work. Because the experimental error is small with respect to the ten+ percent adequacy criterion, and is independent of modeling errors, it is neglected. Table 5 lists the experimental natural frequencies and a description of the validation modes with the blind prediction of the 250,000 node Salinas model and percentage difference. The breathing mode (0,0), which was supposed to be one of the validation modes, was not found, because it was out of the frequency band. All blind predictions of the extracted validation modal frequencies are well within the ten+ percent adequacy criteria, therefore the three layer modeling approach was found to be valid. Several other non validation modes are compared in Table 6.

Table 5 - Validation Modal Frequency Comparisons on Closed Cone Shell

Mode Description	Model Frequency (Hz)	Test Frequency (Hz)	% difference
Ovaling N 2-0	589	581	1.38%
Ovaling N 2-0	589	588	0.17%
First Bending	1627.6	1647	-1.18%
First Bending	1627.6	1647	-1.18%
Ovaling N 4-0	2344.1	2372	-1.18%
First Axial	3139.7	3128	0.37%

Table 6 - Non- Validation Modes Frequency Comparison for Closed Cone Shell

Mode Description	Model Frequency (Hz)	Test Frequency (Hz)	% difference
Ovaling N 3-0	1355.8	1348	0.58%
Ovaling N 3-0	1355.9	1350	0.44%
Ovaling N 2-1	1604.4	1653	-2.94%
Ovaling N 2-1	1604.4	1670	-3.93%
Ovaling N 3-1	2006.9	2139	-6.18%
Ovaling N 3-1	2006.9	2141	-6.26%
First Torsion	2395	2317	3.37%
Ovaling N 4-1	2987.1	3187	-6.27%
Second Bending	2889.5	3136	-7.86%
Second Bending	2889.5	3142	-8.04%

8.0 CONCLUSIONS

This model validation case study was performed to validate a modeling technique for a three layer conical shell with a specified number of 20 node solid elements through each layer of the thickness and a simplification of orthotropic material to isotropic for certain known environments. Modal frequencies were selected as response features for specific modes of validation hardware that would exercise the physics of interest. Adequacy criteria were established for comparing a blind model prediction the response features with a modal test. Although some typical industry specifications have a somewhat arbitrary requirement of agreement within five percent on frequencies, the model validation team relaxed these requirements to $\pm(10^2 + U_m^2)^{1/2}$ percent because the shell hardware will be connected to other subassemblies that will have significant impact on the global stiffness. These parallel connections overwhelm small errors in the shell modeling. This rationale is based on developing

adequacy criteria that are as lenient as possible so that a useful model will not be declared invalid (a typical problem found in previous validation work).
Before the validation prediction was made, several correlation and calibration exercises were performed. One calibration exercise focused specifically on identifying two isotropic material constants for the middle bond layer. Experiments and model calibrations were performed to infer these two constants, and multiple samples allowed statistics to be calculated for these material constants. Several model-based correlation studies determined the uncertainty associated with manufacturing tolerances, acceptable manufacturing bond void tolerances, isotropic vs. orthotropic material property assumptions and a code to code comparison of eigenvalues between MSC/Nastran and Salinas. All of the model-based uncertainty quantification indicated that less than one percent error would be introduced by any of these factors in the key modal frequencies.

A correlation between a FE model of a conical-frustum shell and a modal test on this non-validation hardware were utilized to attempt to uncover unknown modeling errors or poor assumptions. The initial correlation indicated that there was some unknown error or errors in the model that were significant, since the lowest ovaling mode prediction was 12 percent high. Because of this correlation effort an error in the original calibration process, and an over simplification in the model form was discovered. Once these were corrected, the blind validation prediction was performed for the different validation hardware on which another modal test had been performed. The blind prediction frequencies were well within the adequacy criteria. One mode that was specified for the response features could not be extracted in the modal test because it was too high in frequency. Since all other validation frequencies and many non-validation frequencies were well within the adequacy criteria, the three layer modeling approach was declared valid.

This paper emphasizes developing a team which specifies a rationale for validation in advance. The importance of the correlation, uncertainty quantification and calibration efforts were paramount to confirm adequate modeling form and reduce huge uncertainties in certain material property constants. Confidence in the modeling approach resulted from this effort.

9.0 REFERENCES

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